Psychology 465A
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Markovian Observer

1. Kubovy Rapoport & Tversky (1970, *P&P*) showed that in numerical signal detection (stimuli are *numbers* sampled from overlapping probability distributions) subjects did not use single criterion value (which in this case would be a number) for all trials of a given run. Sometimes as many as 7% violations of SDT single criterion assmption.

2. Ward (1973, *P&P*) observed (as others had) that the sequence of judgment trials is a first order Markov process with transition matrix as below:

		Stimulus distribution on Trial N		
		1	**2**	**Sum**
Stimulus Distribution on Trial N-1	**1**	p	q	1.0
	2	r	s	1.0

and asymptotic state probabilities $P(1) = q/(r+q)$, $P(2) = r/(r+q)$. Ward postulated that subjects might be sensitive to the transition probabilities (*p, q, r, s*) rather than the asymptotic state probabilities when setting their criteria.

3. Consider the following transition matrices:

Alternations		Random		Runs	
.1	.9	.5	.5	.9	.1
.9	.1	.5	.5	.1	.9

Ward (1973) showed that subjects indeed were sensitive to the transition probabilities and produced higher *d'* for alternation and runs conditions than for random condition. Moreover, *d'* in the random condition was lower than the nominal *d'*=1, presumably because of violations of the single criterion rule in that condition.

4. Ward, Livingston and Li (1988) extended signal detection theory to the consideration of the case where the sequence of trials is a first Markov process. For the (so-called) Markovian Observer the usual expected-value decision rule (maximizes expected value),

$$\text{if } x_n \begin{Bmatrix} \geq \\ < \end{Bmatrix} \beta \text{ then say } \begin{Bmatrix} \mathbf{2} \\ \mathbf{1} \end{Bmatrix}$$

where

$$\beta = \frac{P(1)}{P(2)}\left(\frac{V_{cr} + V_{miss}}{V_{hit} + V_{fa}}\right)$$

is replaced by a decision rule conditional on the stimulus distribution on Trial N-1 (S_{n-1}) in which *P(1)* and *P(2)* are replaced by *p,q* and *r,s* respectively, as follows:

$$S_{n-1} = \mathbf{1}: if\ x_n \begin{Bmatrix} \geq \\ < \end{Bmatrix} \beta_1\ say \begin{Bmatrix} \mathbf{2} \\ \mathbf{1} \end{Bmatrix}$$

$$S_{n-1} = \mathbf{2}: if\ x_n \begin{Bmatrix} \geq \\ < \end{Bmatrix} \beta_2\ say \begin{Bmatrix} \mathbf{2} \\ \mathbf{1} \end{Bmatrix}$$

where $\beta_1 = \frac{p}{q}\left(\frac{V_{cr} + V_{miss}}{V_{hit} + V_{fa}}\right)$ and $\beta_2 = \frac{r}{s}\left(\frac{V_{cr} + V_{miss}}{V_{hit} + V_{fa}}\right)$.

5. Ward et al (1988) also defined an Ignorant Markovian Observer that didn't know *p,q,r,s* and had to estimate them from the observed transitions in the sequence of trials, e.g. in the following sequence of samples from distributions 1 and 2
112122222222112121111222221112211111122222
the estimates of *p,q,r,s* are: $\hat{p} = 11/17 \approx 0.65$, $\hat{q} = 1 - \hat{p} = 0.35$, $\hat{r} = 16/22 \approx 0.73$, $\hat{s} = 1 - \hat{r} = 0.27$. Various IMOs were simulated, with the parameter being from how far back in the trial sequence the estimates were based upon, from all of it (Bayesian) to just the last few trials (limited short term memory). We found we could explain the data if the estimates were based on only the last 7-15 trials (varied over subjects), consistent with typical limits on STM. This performance allows improvements on nominal d' when there is information in the sequence of trials (the transition matrix is not uniform), but, ironically, predicts worsened performance in the random case caused by the estimates jumping around because of the limited basis, causing the criteria to jump around and decrease *d'*.